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Jefferson Lab

EXPLORING THE NATURE OF MATTER

# Spin Sum Rules and 3D Nucleon Structure (6/6)


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UNIVERSITÉ PARIS SUD

Comprendre le monde, construire l'avenir®

Université de Liège

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# Outline

## Lecture 1

- Introduction
- Tour in phase space
- Galileo *vs* Lorentz

## Lecture 2

- Photon point of view
- Galileo *vs* Lorentz : round 2
- Nucleon 1D picture

## Lecture 3

- Nucleon 2D picture
- 1D+2D=3D
- Galileo *vs* Lorentz : round 3

## Lecture 4

- Another nucleon 3D picture
- Tour in Fock space
- 3D+3D=... 5D !

## Lecture 5

- Canonical *vs* kinetic
- Free fall in gauge space
- Physical interpretation

## Lecture 6

- OAM : a matter of path
- Phase space *vs* energy-momentum
- Summary

## Textbook

### Starting point

$$\mathcal{L} = \mathcal{L}[\phi, \partial_\mu \phi] \qquad \phi = \psi, A_\nu$$

**Lagrangian** **Dynamical variables**

$$\psi(x) \mapsto U(x)\psi(x)$$
$$A_\mu(x) \mapsto U(x)\left[A_\mu(x) + \tfrac{i}{g}\partial_\mu\right]U^{-1}(x)$$

### Conserved tensors

**Energy-momentum**

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\,\partial^\nu \phi - g^{\mu\nu}\mathcal{L}$$

**Translation invariance**

**Generalized angular momentum**

$$M^{\mu\nu\rho} = -i\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\,S^{\nu\rho}\phi + (x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu})$$

**Lorentz invariance**

**Gauge non-invariant !**

# Canonical formalism

## Gauge covariant

### Starting point

$$\mathcal{L} = \mathcal{L}[\phi, D^{\text{pure}}_\mu \phi] \qquad \phi = \psi, A^{\text{phys}}_\nu$$

$$\psi(x) \mapsto U(x)\psi(x)$$
$$A^{\text{phys}}_\mu(x) \mapsto U(x) A^{\text{phys}}_\mu(x) U^{-1}(x)$$

**Lagrangian** **Dynamical variables**

### Conserved tensors

**Energy-momentum**

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(D^{\text{pure}}_\mu \phi)} D^\nu_{\text{pure}}\phi - g^{\mu\nu}\mathcal{L}$$

**Translation invariance**

**Generalized angular momentum**

$$M^{\mu\nu\rho} = -i \frac{\partial \mathcal{L}}{\partial(D^{\text{pure}}_\mu \phi)} S^{\nu\rho}\phi + (x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu})$$

**Lorentz invariance**

**Gauge invariant !**

**[C.L. (2013)]**

## Gauge invariant

### Starting point

$$\mathcal{L} = \mathcal{L}[\hat{\phi}, \partial_\mu \hat{\phi}]$$

Lagrangian

$$\hat{\phi} = \hat{\psi}, \hat{A}_\nu^{\text{phys}}$$

Dynamical variables

Dirac variables

Dressing field

$$\hat{\psi}(x) \equiv U_{\text{pure}}^{-1}(x)\psi(x)$$

$$\hat{A}_\mu^{\text{phys}}(x) \equiv U_{\text{pure}}^{-1}(x)\left[A_\mu(x) + \tfrac{i}{g}\partial_\mu\right] U_{\text{pure}}(x)$$

$$U_{\text{pure}}(x) \mapsto U(x)U_{\text{pure}}(x)$$

[Dirac (1955)]
[Mandelstam (1962)]

### Conserved tensors

Energy-momentum

$$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\hat{\phi})}\,\partial^\nu\hat{\phi} - g^{\mu\nu}\mathcal{L}$$

Translation invariance

Generalized angular momentum

$$M^{\mu\nu\rho} = -i\frac{\partial\mathcal{L}}{\partial(\partial_\mu\hat{\phi})}\,S^{\nu\rho}\hat{\phi} + (x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu})$$

Lorentz invariance

Gauge invariant !

[Chen (2012)]
[C.L. (2013)]

# OAM and path dependence



[Ji, Xiong, Yuan (2012)]
[Hatta (2012)]
[C.L. (2013)]

**Quark generalized OAM operator**

$$\boxed{L^{\mu\nu\rho}_{q,\mathcal{C}}(x) = \overline{\psi}(x)\gamma^\mu(x^\nu\,\tfrac{i}{2}\overleftrightarrow{D}^\rho_{\text{pure}} - x^\rho\,\tfrac{i}{2}\overleftrightarrow{D}^\nu_{\text{pure}})\psi(x)}$$

# OAM and path dependence

[Ji, Xiong, Yuan (2012)]
[Hatta (2012)]
[C.L. (2013)]

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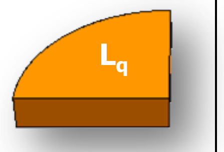
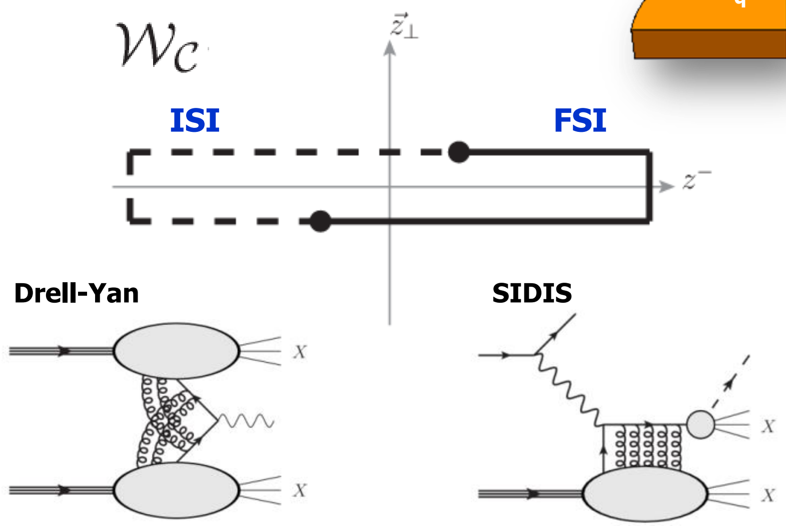
**Light-front**




$D^{\text{pure}}_\mu \stackrel{A^+=0}{=} \partial_\mu$ $\boxed{\ell_z^{\text{DY}} = \ell_z^{\text{SIDIS}}}$ **Naive T-even**

# OAM and path dependence

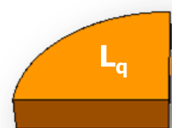


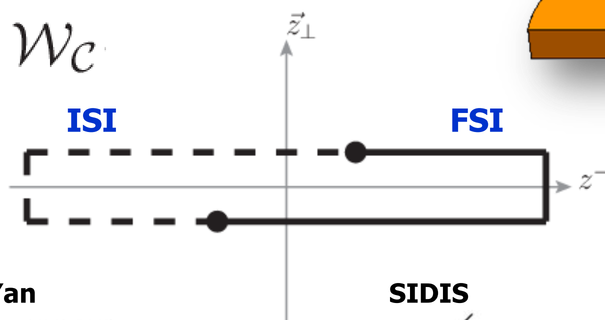
[Ji, Xiong, Yuan (2012)]
[Hatta (2012)]
[C.L. (2013)]

**Quark generalized OAM operator**

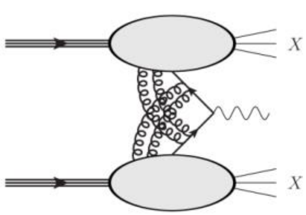
$$\boxed{L^{\mu\nu\rho}_{q,\mathcal{C}}(x)=\overline{\psi}(x)\gamma^\mu(x^\nu\,\tfrac{i}{2}\overleftrightarrow{D}^\rho_{\text{pure}}-x^\rho\,\tfrac{i}{2}\overleftrightarrow{D}^\nu_{\text{pure}})\psi(x)}$$

## Light-front

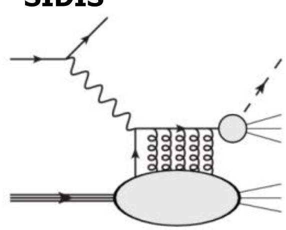



**Drell-Yan**




**SIDIS**

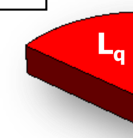


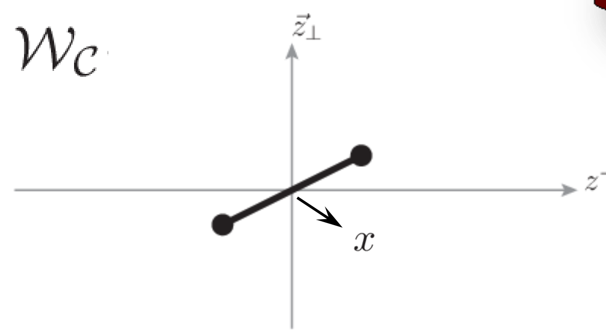

$D^{\text{pure}}_\mu \overset{A^+=0}{=} \partial_\mu$

$\boxed{\ell_z^{\text{DY}}=\ell_z^{\text{SIDIS}}}$

**Naive T-even**

## *x*-based Fock-Schwinger




**Coincides *locally* with kinetic quark OAM**

$$A_\mu(x)=A^{\text{pure}}_\mu(x)$$

$$A_\mu(y)\neq A^{\text{pure}}_\mu(y)\qquad y\neq x$$

$$L^{\mu\nu\rho}_q(x)=\overline{\psi}(x)\gamma^\mu(x^\nu iD^\rho-x^\rho iD^\nu)\psi(x)$$

# Phase-space approach

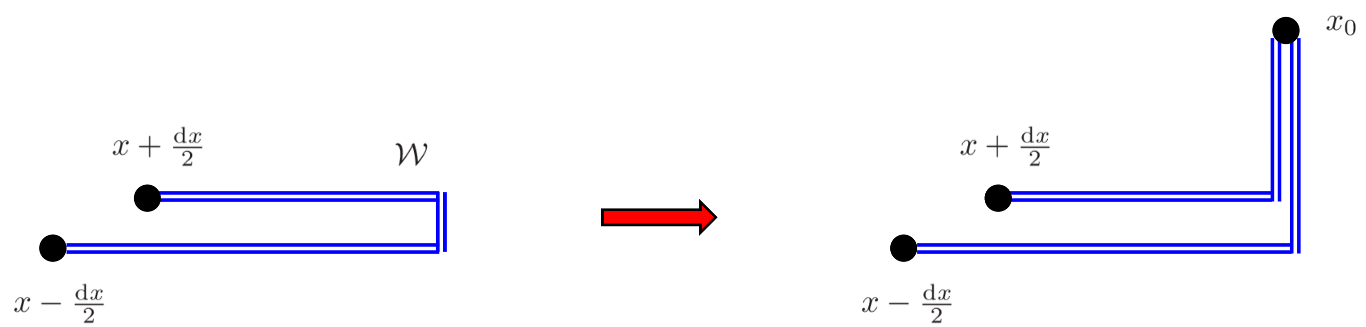


**Quark generalized OAM operator**

$$L^{\mu\nu\rho}_{q,\mathcal{C}}(x) = x^\nu T^{\mu\rho}_{q,\mathcal{C}}(x) - x^\rho T^{\mu\nu}_{q,\mathcal{C}}(x)$$

**Quark energy-momentum tensor operator**

$$\begin{aligned} T^{\mu\nu}_{q,\mathcal{C}}(x) &= \overline{\psi}(x)\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu_{\text{pure}}\psi(x) \\ &= \overline{\psi}(x)\mathcal{W}_{xx_0}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{\partial}^\nu\mathcal{W}_{x_0x}\psi(x) \end{aligned}$$




**[C.L., Pasquini, Xiong, Yuan (2012)]**
**[C.L. (2013)]**

**Quark generalized OAM operator**

$$L^{\mu\nu\rho}_{q,\mathcal{C}}(k,x) = x^\nu T^{\mu\rho}_{q,\mathcal{C}}(k,x) - x^\rho T^{\mu\nu}_{q,\mathcal{C}}(k,x)$$

**Quark energy-momentum tensor operator**

$$\begin{aligned}
T^{\mu\nu}_{q,\mathcal{C}}(k,x) &= \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \overline{\psi}(x_f)\gamma^\mu \left(\mathcal{W}_{x_f x_i} \tfrac{i}{2}\overrightarrow{D}^\nu_{\text{pure}} - \tfrac{i}{2}\overleftarrow{D}^\nu_{\text{pure}}\mathcal{W}_{x_f x_i}\right)\psi(x_i) \\
&= \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \overline{\psi}(x_f)\mathcal{W}_{x_f x_0}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{\partial}^\nu \mathcal{W}_{x_0 x_i}\psi(x_i) \\
&= \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, i\partial^\nu_z \left[\overline{\psi}(x_f)\gamma^\mu \mathcal{W}_{x_f x_i}\psi(x_i)\right] \\
&= k^\nu \underbrace{\int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \overline{\psi}(x_f)\gamma^\mu \mathcal{W}_{x_f x_i}\psi(x_i)}_{\widehat{W}^{[\gamma^\mu]}_{\mathcal{C}}(k,x)}
\end{aligned}$$

$$x_i = x + \tfrac{z}{2}$$
$$x_f = x - \tfrac{z}{2}$$

[C.L., Pasquini, Xiong, Yuan (2012)]
[C.L. (2013)]

# Phase-space approach

**Quark generalized OAM operator**

$$L^{\mu\nu\rho}_{q,\mathcal{C}}(k,x) = (x^\nu k^\rho - x^\rho k^\nu)\, \widehat{W}^{[\gamma^\mu]}_{\mathcal{C}}(k,x)$$

**Quark average OAM**

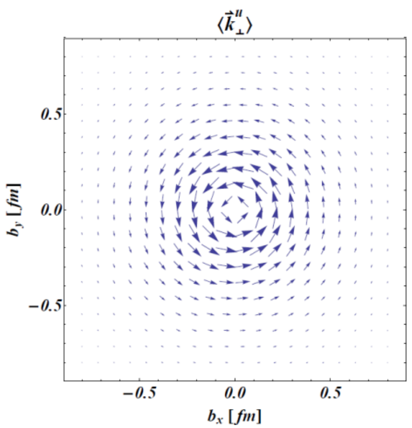
$$\ell^q_z = \int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \mathrm{d}^2b_\perp\, \ell^q_z(x,\vec{k}_\perp,\vec{b}_\perp)$$

**Quark phase-space density of OAM**

$$\begin{aligned}\ell^q_z(x,\vec{k}_\perp,\vec{b}_\perp) &= \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, \langle P^+, \tfrac{\vec{\Delta}_\perp}{2}|L^{+12}_{q,\mathcal{C}}(xP^+,\vec{k}_\perp,0,\vec{b}_\perp,0)|P^+,-\tfrac{\vec{\Delta}_\perp}{2}\rangle\\ &= (\vec{b}_\perp\times\vec{k}_\perp)_z\, \rho^{[\gamma^+]}_W(x,\vec{k}_\perp,\vec{b}_\perp)\end{aligned}$$

[C.L., Pasquini, Xiong, Yuan (2012)]
[C.L. (2013)]

### Quark momentum-space density of OAM




$$\begin{aligned}
\ell^q_z(x,\vec{k}_\perp) &= \int \mathrm{d}^2b_\perp\, \ell^q_z(x,\vec{k}_\perp,\vec{b}_\perp) \\
&= \int \mathrm{d}^2b_\perp\, (\vec{b}_\perp \times \vec{k}_\perp)_z\, \rho_W^{[\gamma^+]}(x,\vec{k}_\perp,\vec{b}_\perp) \\
&= (\vec{k}_\perp \times i\vec{\nabla}_{\Delta_\perp})_z\, W^{[\gamma^+]}_{++}(x,0,\vec{k}_\perp,\vec{\Delta}_\perp)\Big|_{\vec{\Delta}_\perp=\vec{0}_\perp} \\
&= -\frac{\vec{k}^2_\perp}{M^2}\, F_{14}(x,0,\vec{k}_\perp,\vec{0}_\perp)
\end{aligned}$$

**Parametrization** $$W^{[\gamma^+]}_{\Lambda'\Lambda} = \frac{1}{2M}\,\overline{u}(p',\Lambda')\left[F_{11} + \frac{i\sigma^{k_\perp+}}{P^+}F_{12} + \frac{i\sigma^{\Delta_\perp+}}{P^+}F_{13} + \frac{i\sigma^{k_\perp\Delta_\perp}}{M^2}F_{14}\right]u(p,\Lambda)$$

**[Meißner, Metz, Schlegel (2009)]**

$$\boxed{\ell^q_z = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}^2_\perp}{M^2}\,F_{14}(x,0,\vec{k}_\perp,\vec{0}_\perp)}$$

- **No known physical process !**
- **Can (and will) be calculated on the lattice**

**[C.L., Pasquini, Xiong, Yuan (2012)]**
**[C.L. (2013)]**

# Ji's approach



**Let's focus on the energy-momentum tensor !**

Energy density

Momentum density

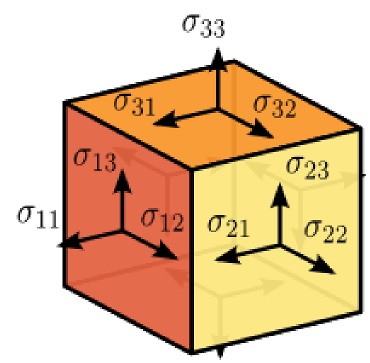
$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress

Normal stress (pressure)

Energy flux

Momentum flux




**In rest frame**

$$M = \int \mathrm{d}^3 r \, T^{00}(\vec{r})$$

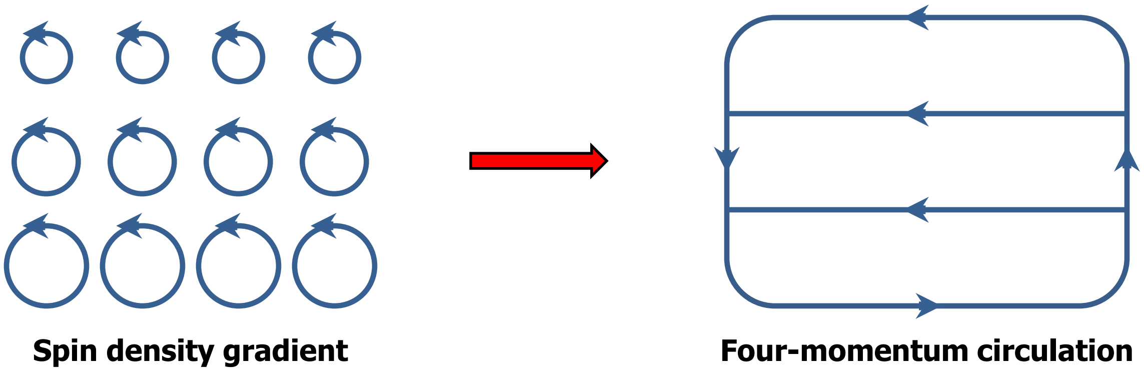
$$L^i = \int \mathrm{d}^3 r \, \epsilon^{ijk} r^j T^{0k}(\vec{r})$$

# Ji's approach

**In presence of spin density** $T^{0i} \neq T^{i0}$

**Belinfante « improvement »**

$$\begin{aligned} T_B^{\mu\nu} &\equiv T^{\mu\nu} + \tfrac{1}{2}\partial_\lambda[S^{\lambda\mu\nu} + S^{\mu\nu\lambda} + S^{\nu\mu\lambda}] \\ &= T_B^{\nu\mu} \end{aligned}$$




**In rest frame**

$$M = \int \mathrm{d}^3 r\, T_B^{00}(\vec{r})$$

$$J^i = \int \mathrm{d}^3 r\, \epsilon^{ijk} r^j T_B^{0k}(\vec{r})$$

**No explicit « spin » contribution !**

**Energy-momentum operator**

$$T_B^{\mu\nu} = \tfrac{1}{2}\,\overline{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi - 2\,\mathrm{Tr}[F^{\mu\alpha}F^{\nu}{}_{\alpha}] + \tfrac{1}{2}\,g^{\mu\nu}\,\mathrm{Tr}\left[F^{\alpha\beta}F_{\alpha\beta}\right]$$

$$a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

**Matrix elements**

**Normalization**

$$\langle p', \vec{s}|p, \vec{s}\rangle = 2P^0\,(2\pi)^2\,\delta^{(3)}(\vec{p}' - \vec{p})$$

$$\begin{aligned}\langle\langle \textstyle\int \mathrm{d}^3r\, O(\vec{r})\rangle\rangle &\equiv \frac{\langle P, \vec{s}|\int \mathrm{d}^3r\, O(\vec{r})|P, \vec{s}\rangle}{\langle P, \vec{s}|P, \vec{s}\rangle} \\ &= \frac{1}{2P^0}\,\langle P, \vec{s}|O(\vec{0})|P, \vec{s}\rangle\end{aligned}$$

$$\begin{aligned}\langle\langle \textstyle\int \mathrm{d}^3r\, \vec{r}\, O(\vec{r})\rangle\rangle &\equiv \lim_{\vec{\Delta}\to\vec{0}} \frac{\langle p', \vec{s}|\int \mathrm{d}^3r\, \vec{r}\, O(\vec{r})|p, \vec{s}\rangle}{\langle p', \vec{s}|p, \vec{s}\rangle} \\ &= \frac{1}{2P^0}\left[-i\vec{\nabla}_\Delta\langle p', \vec{s}|O(\vec{0})|p, \vec{s}\rangle\right]_{\vec{\Delta}=\vec{0}}\end{aligned}$$

**Energy-momentum FFs**

$$\langle p'|T_B^{\mu\nu}(0)|p\rangle = \overline{u}(p')\left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2}\,A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{4M}\,B(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}\,C(t)\right]u(p)$$

**Momentum sum rule**

$$\langle\langle \textstyle\int \mathrm{d}^3r\, T_B^{0\mu}(\vec{r})\rangle\rangle = A(0)\,P^\mu \quad \Longrightarrow \quad \boxed{A(0)=1}$$

**Angular momentum sum rule**

$$\langle\langle \textstyle\int \mathrm{d}^3r\, \vec{J}_B(\vec{r})\rangle\rangle = \frac{\vec{s}}{2}\left[A(0) + \frac{P^0}{M}\,B(0)\right] - \frac{(\vec{P}\cdot\vec{s})\vec{P}}{2M(P^0+M)}\,B(0)$$

$P^\mu = (P^0, 0, 0, P_z)$

$\vec{s} = (0,0,1)$

$$\langle\langle \textstyle\int \mathrm{d}^3r\, J_B^z(\vec{r})\rangle\rangle = \frac{1}{2}\left[A(0)+B(0)\right] \quad \Longrightarrow \quad \boxed{A(0)+B(0)=1}$$

$$\Longrightarrow \quad \boxed{B(0)=0}$$

**Vanishing gravitomagnetic moment !**

[Ji (1997)]

**Energy-momentum FFs**

$$\boxed{\sum_{q,G}\overline{C}_{q,G}(t)=0}$$

$$\langle p'|T^{\mu\nu}_{Bq,G}(0)|p\rangle=\overline{u}(p')\left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2}\,A_{q,G}(t)+\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{4M}\,B_{q,G}(t)\right.$$
$$\left.+\frac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{M}\,C_{q,G}(t)+\underbrace{M g^{\mu\nu}\,\overline{C}_{q,G}(t)}_{\text{Non-conserved current}}\right]u(p)$$

**Momentum sum rule**

$$\langle\langle\textstyle\int\mathrm{d}^3r\,T^{0\mu}_{Bq,G}(\vec{r})\rangle\rangle=A_{q,G}(0)\,P^\mu \quad\Longrightarrow\quad \boxed{\sum_{q,G}A_{q,G}(0)=1}$$

**Angular momentum sum rule**

$$\langle\langle\textstyle\int\mathrm{d}^3r\,\vec{J}_{Bq,G}(\vec{r})\rangle\rangle=\frac{\vec{s}}{2}\left[A_{q,G}(0)+\frac{P^0}{M}\,B_{q,G}(0)\right]-\frac{(\vec{P}\cdot\vec{s})\vec{P}}{2M(P^0+M)}\,B_{q,G}(0)$$

$P^\mu=(P^0,0,0,P_z)$

$\vec{s}=(0,0,1)$

$$\langle\langle\textstyle\int\mathrm{d}^3r\,J^z_{Bq,G}(\vec{r})\rangle\rangle=\frac{1}{2}\left[A_{q,G}(0)+B_{q,G}(0)\right] \quad\Longrightarrow\quad \boxed{\sum_{q,G}\left[A_{q,G}(0)+B_{q,G}(0)\right]=1}$$

$$\Longrightarrow\quad \boxed{\sum_{q,G}B_{q,G}(0)=0}$$

**Vanishing gravitomagnetic moment !**

**[Ji (1997)]**

**Leading-twist component**

$$\langle p'|T^{++}_{Bq}(0)|p\rangle = \overline{u}(p')P^+\gamma^+u(p)\left[A_q(t)+4\xi^2\,C_q(t)\right] + \overline{u}(p')P^+\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\,u(p)\left[B_q(t)-4\xi^2\,C_q(t)\right]$$

**Link with GPDs**

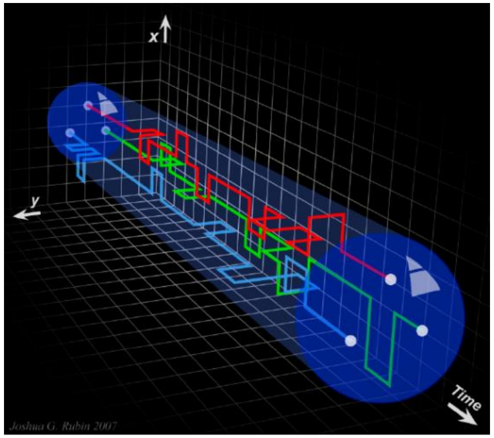
$$\begin{aligned}\langle p'|\overline{\psi}(0)\gamma^+iD^+\psi(0)|p\rangle &= 2(P^+)^2\int \mathrm{d}x\,x\left[\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ixP^+z^-}\,\langle p'|\overline{\psi}(-\tfrac{z^-}{2})\gamma^+\psi(\tfrac{z^-}{2})|p,\rangle\right]\\ &= \overline{u}(p')P^+\gamma^+u(p)\int\mathrm{d}x\,x\,H_q(x,\xi,t)\\ &\quad+\overline{u}(p')P^+\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\,u(p)\int\mathrm{d}x\,x\,E_q(x,\xi,t)\end{aligned}$$

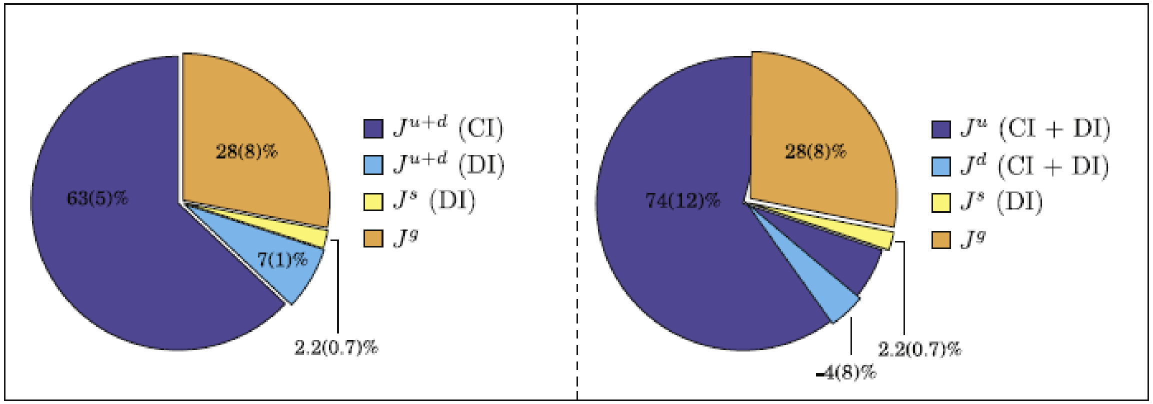
$$\Longrightarrow\quad \boxed{\langle\langle\textstyle\int \mathrm{d}^3r\,J^z_{Bq}(\vec{r})\rangle\rangle = \frac{1}{2}\displaystyle\int\mathrm{d}x\,x\,[H_q(x,0,0)+E_q(x,0,0)]}$$

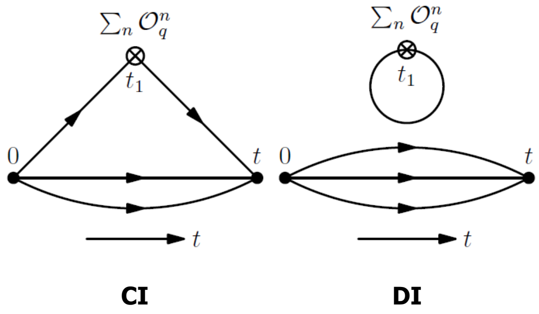
**Measurable *e.g.* in DVCS !**

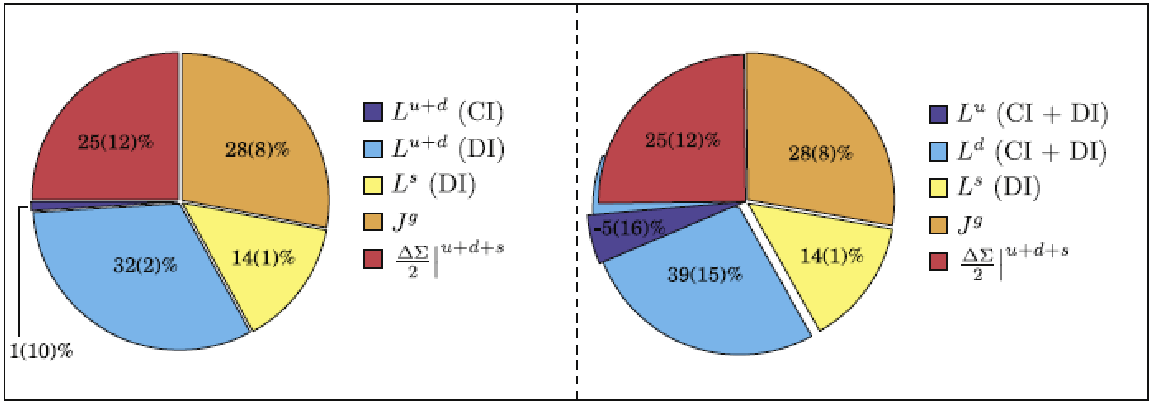
**[Ji (1997)]**

# Lattice results




[Deka *et al.* (2013)]

# Kinetic *vs* canonical OAM



**Kinetic OAM (Ji)**

$$L_z = \underbrace{\frac{1}{2}\int \mathrm{d}x\, x[H(x,0,0)+E(x,0,0)]}_{J_z} - \underbrace{\frac{1}{2}\int \mathrm{d}x \tilde{H}(x,0,0)}_{S_z}$$

[Ji (1997)]

$$= -\underbrace{\int \mathrm{d}x\, x\, G_2(x,0,0)}_{\text{Pure twist-3}} = \int \mathrm{d}x\, x\,[H(x,0,0)+E(x,0,0)+\tilde{E}_{2T}(x,0,0)]$$

[Penttinen *et al.* (2000)]
[Kiptily, Polyakov (2004)]
[Hatta (2012)]

**Quark *naive* canonical OAM (Jaffe-Manohar)**

$$\mathcal{L}_z = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\vec{k}_\perp^2}{2M^2} h_{1T}^\perp(x,\vec{k}_\perp)$$

⚠ **Model-dependent !**

[Burkardt (2007)]
[Efremov *et al.* (2008,2010)]
[She, Zhu, Ma (2009)]
[Avakian *et al.* (2010)]
[C.L., Pasquini (2011)]

**Canonical OAM (Jaffe-Manohar)**

$$\ell_z = -\int \mathrm{d}x\, \mathrm{d}^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x,0,\vec{k}_\perp,\vec{0}_\perp)$$

[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (2012)]
[Hatta (2012)]
[Kanazawa *et al.* (2014)]

| Model | LCCQM | | | $\chi$QSM | | |
|---|---|---|---|---|---|---|
| $q$ | $u$ | $d$ | Total | $u$ | $d$ | Total |
| $\ell_z^q$ | 0.131 | −0.005 | 0.126 | 0.073 | −0.004 | 0.069 |
| $L_z^q$ | 0.071 | 0.055 | 0.126 | −0.008 | 0.077 | 0.069 |
| $\mathcal{L}_z^q$ | 0.169 | −0.042 | 0.126 | 0.093 | −0.023 | 0.069 |

⚠ **No gluons and not QCD EOM !**

$\ell_z = L_z$ **but** $\ell_z^q \neq L_z^q$

[C.L., Pasquini (2011)]

# GPDs *vs* TMDs



## Multipole structure of the amplitudes

Monopole Dipole Quadrupole

**TMDs**

| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $f_1$ | $\frac{k_y}{M}\, h_1^\perp$ | $-\frac{k_x}{M}\, h_1^\perp$ | |
| $T_x$ | $\frac{k_y}{M}\, f_{1T}^\perp$ | $h_1 + \frac{k_x^2-k_y^2}{2M^2}\, h_{1T}^\perp$ | $\frac{k_x k_y}{M^2}\, h_{1T}^\perp$ | $\frac{k_x}{M}\, g_{1T}$ |
| $T_y$ | $-\frac{k_x}{M}\, f_{1T}^\perp$ | $\frac{k_x k_y}{M^2}\, h_{1T}^\perp$ | $h_1 - \frac{k_x^2-k_y^2}{2M^2}\, h_{1T}^\perp$ | $\frac{k_y}{M}\, g_{1T}$ |
| $L$ | | $\frac{k_x}{M}\, h_{1L}^\perp$ | $\frac{k_y}{M}\, h_{1L}^\perp$ | $g_{1L}$ |

**GPDs** $\xi = 0$

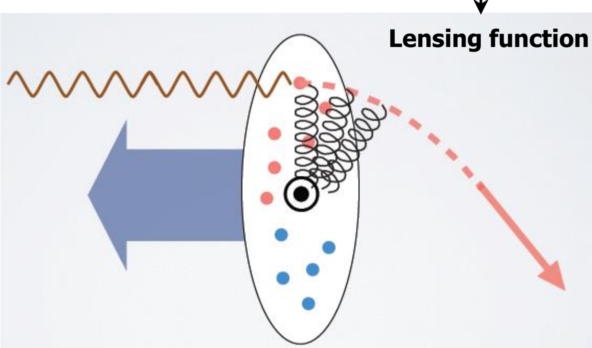
| | $U$ | $T_x$ | $T_y$ | $L$ |
|---|---|---|---|---|
| $U$ | $H$ | $i\,\frac{\Delta_y}{2M}\,(2\tilde{H}_T + E_T)$ | $-i\,\frac{\Delta_x}{2M}\,(2\tilde{H}_T + E_T)$ | |
| $T_x$ | $i\,\frac{\Delta_y}{2M}\,E$ | $H_T - \frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$ | $-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$ | |
| $T_y$ | $-i\,\frac{\Delta_x}{2M}\,E$ | $-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$ | $H_T + \frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$ | |
| $L$ | | | | $\tilde{H}$ |

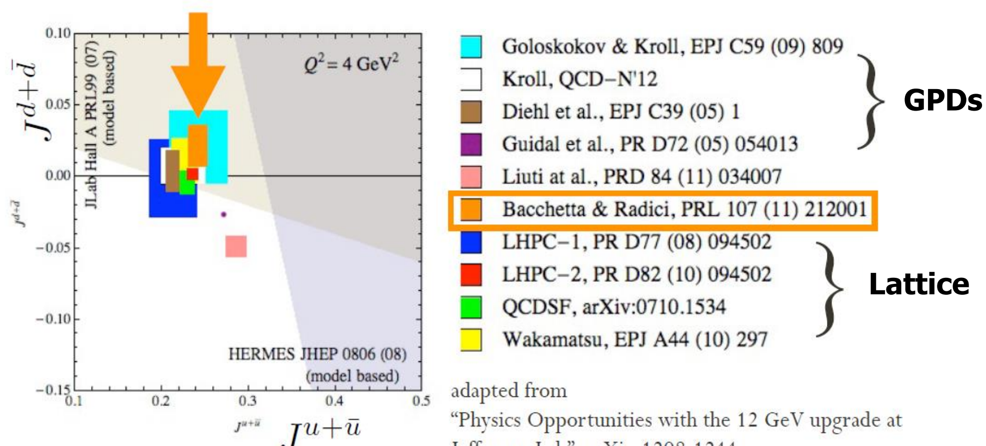
$$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$$

## Phenomenological relation

$$\int \mathrm{d}^2k_\perp\, f_{1T}^{\perp q}(x,\vec{k}_\perp;\mu^2) = -L(x)E_q(x,0,0;\mu^2)$$

**Lensing function**





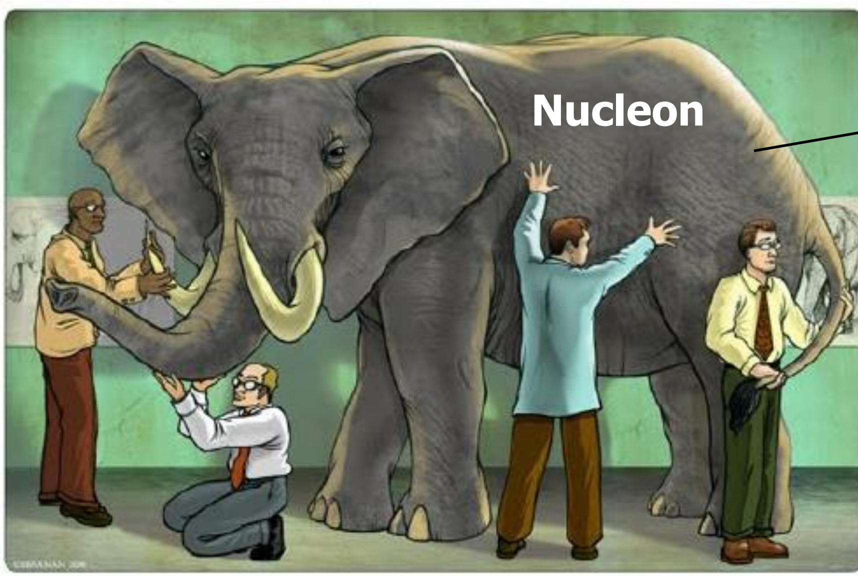

adapted from "Physics Opportunities with the 12 GeV upgrade at Jefferson Lab", arXiv:1208.1244

**[Burkardt (2002)]**
**[Bacchetta, Radici (2011)]**

# Summary

## Lectures 1-6

- **Phase-space/Wigner distributions can be defined for quark & gluons**
- **Only <u>projections</u> of these distributions are experimentally accessible**
- **Nucleon spin decomposition is not unique (so be consistent !)**
- **Quark and gluon OAM account for ~50% of nucleon spin**
- **Spin and OAM are at the origin of observed asymmetries**




**GPDs TMDs FFs PDFs**

**LFWFs**